

OGAWA Hidemitsu “Advancing Mathematical Sciences via Development of the Theory of Pseudobiorthogonal Bases”

1. BIOGRAPHY

Born on January 17, 1942

- 1965 Awarded B.Eng., **Tokyo Institute of Technology**
- 1977 Awarded Ph.D. (Eng.), **Tokyo Institute of Technology**
- 1965 Appointed Researcher at **Electrotechnical Laboratory, Ministry of International Trade and Industry (MITI)**
- 1972 Appointed Assistant Professor at **Tokyo Institute of Technology**; Associate Professor (1978), Professor (1987), Dean of the Graduate School of Information Science and Engineering (2001), Professor Emeritus (2005)
- 2005 Appointed Adviser at **Toray Engineering Co., Ltd.**
- 2007 Appointed Professor at **Tokyo University of Social Welfare**
- 2011 Appointed Director, **The Matsumae International Foundation**



MAIN AWARDS Best Paper Awards (1976, 1985, 1990, 1993, 1994, shared) and Yonezawa Memorial Paper Award (1993, shared) from **The Institute of Electronics, Information and Communication Engineers (IEICE)**. Outstanding Technology Award (2001, shared) from **The Society of Instrument and Control Engineers (SICE)**. Technology Award (2011, shared) from **The Japan Society for Precision Engineering (JSPE)**. Honorary Doctor (1994) from the **Lappeenranta University of Technology, Finland**.

2. SUMMARY

Ogawa Hidemitsu has made several fundamental contributions to advancing mathematical sciences. His contributions range from core **fundamental** reformulations, unifying several difficult to understand concepts in sampling and learning theory to developments of more efficient and provably correct **algorithms** in machine learning, leading to results that help with **translation** of research innovations to real world impact.

3. RESEARCH ACHIEVEMENT (Fundamental, Algorithmic, Translational)

(1) Creating a new scientific language (Fundamental)

When Albert Einstein started his research on general theory of relativity, he noticed that he did not have the appropriate language to describe his ideas. However, his exposure to ‘differential geometry’ helped him exploit novel tools, leading to his development of the general theory of relativity. Indeed in his Kyoto address of 1922, he famously says “we must search for words before we can

express thoughts”. If we cannot find such a language, we need to create one appropriate for the purpose. Led strongly by this belief, Ogawa created a novel ‘language’ and mathematical framework named the “theory of pseudo biorthogonal bases (PBOB)” for systematically addressing complex concepts in signal and image processing, pioneering its use in the now hugely popular area of machine learning [13].

The PBOB extends the notion of orthonormal basis (ONB) toward three different directions. The first direction is related to the Fourier series expansion, which is an expansion with respect to an ONB. The pseudoorthogonal basis (POB) is an extension of the ONB to the case of linearly dependent system that can be dealt with as the usual ONB. The second direction is related to the power series expansion, which is an expansion with respect to a biorthonormal basis (BONB). The pseudo-biorthogonal basis (PBOB) is an extension of the BONB to the case of linearly dependent system that can be dealt with using tools used for BONB. The third direction is quite innovative. There are situations where the original signals to be processed belong to a subspace of an infinite dimensional space, and elements of a basis do not always belong to the original signal space. The extended pseudo-biorthogonal basis (EPBOB) is an extension of the PBOB to the case of system whose elements do not always belong to the original signal space that can be dealt with using results from PBOB. The term PBOB is also collectively used to refer to these three kinds of bases.

Historically speaking, Iijima Taizo originally introduced the concept of POB for the Euclidean space in 1973 [1]. Ogawa worked this idea into a full-blown theory for a Hilbert space by using functional analysis [2]. Within this mathematical framework, POB has naturally been extended into PBOB and EPBOB. Finally, these results have been published in a book of over 400 pages [13]. The results on POB was the recipient of the Best Paper Award from IEICE.

The theory of PBOB has three kinds of applications. The first one is to exploit the redundant expression of signals to reduce noise in communication and signal processing. It also provides an analog coding theory for non-digital signals, for example, in the context of a CT scanner in the hospital that happens to use analog encoding. By using the theory, we proposed an image reconstruction method that is robust against noise and system failures [3]. This work received the Best Paper Award from IEICE. The second application is to unify different concepts and scenarios. The third application is to use the EPBOB to provide several kinds of rich sampling theorems including ones that cannot be constructed by PBOB alone.

(2) Developing unifying theories across domains (Fundamental)

Ogawa has always strived to develop deeper understanding of problems by looking at whether there are unifying aspects that provide a hitherto unexplored explanation of concepts. He strongly believed that it is the fundamental role of mathematical sciences to provide this unifying vision - both within various instantiations of the problem and across domains. We highlight some examples of these contributions here.

(2.1) Unifying different contexts and situations: A problem has different manifestations depending on the situation. For example, consider sampling theorems. There are sampling theorems for single variable or multi-variate functions; for Nyquist sampling, over sampling or under sampling; for sampling with uniformly or non-uniformly spaced sample points; and for low-pass or band-pass functions. Traditionally, each case tended to be discussed individually. Ogawa developed a generalized sampling theorem in the reproducing kernel Hilbert space (RKHS) which cohesively considered all these situations under a single umbrella framework [4], [13].

(2.2) Unifying different fields of research: We can realize a unified view across domains by identifying and elucidating the common mathematical structure which address the true nature of the problems without being influenced purely by their appearance. As a result, we can effectively deploy a result obtained in one field to another complementary field without having to ‘reinvent the wheel’. Ogawa’s work highlighted that problems of sampling theorems, signal and image restoration, CT image reconstruction, and some forms of machine learning algorithms have the same mathematical structure [5]. As a result, for example, the concept of projection learning has been naturally introduced into the field of machine learning, inspired by the formulation of projection filters for image restoration. The significance of these achievements were acknowledged through two Best Paper Awards from IEICE as well as the IEICE Yonezawa Memorial Paper Award.

(2.3) Unifying theory linking finite and infinite spaces: The Karhunen-Loeve (KL) expansion is a well-known method for representing a sample function of a continuous second-order random process. It has a wide range of applications like pattern recognition and signal processing. It is the expansion with respect to eigenfunctions of the integral equation defined by a stochastic process. In most practical applications, however, only a finite number of sample values of the process are available for the KL expansion. In that case, the integral equation is reduced to a matrix. Ogawa provided conditions and methods by which the exact eigenvalues and eigenfunctions of the original integral equation can be obtained from the matrix [6]. This work was awarded the Tejima Memorial Paper Award from The Tejima Foundation of Industrial Education.

(3) Developing functional analytic theory of learning (Algorithmic)

In the early days of Machine Learning research, artificial neural networks (ANNs) and the problem of representing and learning within these frameworks were mainly discussed via empirical computer simulation because of nonlinear elements in NNs. As the result, the generalization capability of the NNs could not be mathematically guaranteed. Ogawa proposed a theoretical framework that enables to analyze the learning problem more formally. Within the framework, the learning problem is formulated as an optimization problem. It minimizes generalization error given in terms of the norm of the original function space rather than the sampled vector space. He provided exact solutions of the problem and devised the method to design NNs with optimal generalization capabilities [7], [13]. Using this framework, Ogawa and his collaborators provided illuminating answers to

the following problems.

(3.1) Incremental learning: There is a need to look at cases where we have an optimally trained NN and there are additions to the training data to be incorporated at a later stage. Re-learning using the entire training set is a possible but not very efficient solution. An approach to use the intermediate learned result and the additional training data is called incremental learning. A lot of work on incremental learning has been carried out. However, almost all results provide sub-optimal generalization capability compared with batch learning on the entire training set. Ogawa et al. provided an exact incremental learning in the sense that the batch learning on the entire training set would exactly match the incremental solution – a result was awarded the Best Student Paper Award at the 1995 IEEE International Conference on Neural Networks (ICNN) in Perth, Australia and later published as an expanded manuscript in *Neurocomputing* [8].

(3.2) Optimal generalization and objective functions: In the training of NN using the error-backpropagation (BP) algorithm, it has been observed that the error over novel data may increase even though the error over training data decreases. This phenomenon is referred to as overlearning or overfitting. In order to overcome the problem, Ogawa et al. proposed the concept of admissibility of a substitute learning criterion with respect to a ‘true’ learning criterion. The BP criterion can be viewed as a substitute of a ‘true’ criterion, e.g., the projection learning criterion. We obtained necessary and sufficient conditions for the ‘true’ criterion to admit the BP criterion and showed that the conditions can be controlled by sample points [9] – the work being the recipient of the Best Paper Award from IEICE.

(4) Applications to efficient surface profilers (Translational)

White-light interferometry is an elegant technique to measure surface profiles of objects such as semiconductors, liquid-crystal displays, plastic films and precision machinery parts. Many algorithms proposed so far, however, required dense sampling intervals to achieve good accuracy – leading to high computational costs and sample numbers. To reduce it, Ogawa et al. proposed a new surface profiling algorithm by using a generalized sampling theorem [10]. The significantly more efficient algorithm has been installed in a commercial system by Toray Engineering Co., Ltd. that achieved the world’s fastest measurement speed. The result was presented in the program ‘Today’s Close-up’ aired on NHK on July 25, 2000. The result received the Outstanding Technology Award from SICE and Tejima Invention Award from The Tejima Foundation of Industrial Education.

4. SPECIAL REMARKS (Formulation of grand challenges in pattern recognition)

The Hilbert’s twenty-three problems, presented at the International Congress of Mathematicians in 1900, had a decisive influence on the 20th century mathematics. Oka Kiyoshi, who solved three major problems on multivariate complex function theory, said that “Though we can solve problems alone, it is hard to find a good problem. In order to do that, we need collaborative research.” With these examples in mind, Ogawa established the IEICE technical committee addressing problems in

pattern recognition and understanding aimed to guide this field in the 21st century. During two years since April 1991, 33 members discussed appropriate grand challenges worth addressing categorized under four sessions: fundamentals, speech recognition, character and document recognition, and image recognition. The key was discussing what good formulations of the problem are, rather than just looking at how to solve them. We published the results as a book in 1994 [11]. The activity has been taken over in 2007 by the IEICE Technical Committee on Pattern Recognition and Media Understanding.

5. REFERENCES

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- [12] Functional Analysis for Engineering, Morikita Publishing Co., Ltd. (May 2010). (in Japanese)
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